

科目：統計學

系組：統計資訊學系

年級：二

1. (20%)試說明經驗法則(Empirical rule)和柴比雪夫法則(Chebyshev's rule)，並指出兩者間的差異？
2. (20%) 假設統計學考試的時間限定是1小時。為了評估試題的難易情形，教師以10位學生為樣本，在他們繳交試卷的時候記錄了他們考試所花的時間。記錄的時間(近似至最接近的分鐘) 是

33 29 45 60 42 19 52 38 36 40

- a. 計算平均數、中位數與眾數。
 - b. 從(a) 小題所得到的3 個統計量，你得到什麼訊息？
 - c. 請繪製箱型圖(Box-plot)。
 - d. 若有位同學10分鐘即繳交試卷，請計算判斷此同學是否為離群值？
3. (20%)為了檢視公司四次週年廣告促銷的成效，以寄問卷給詢問顧客去年因為促銷而訂購商品的次數。下列資料表列出的機率是從問卷得到的，其中隨機變數 X 代表因促銷活動而促使訂購的次數。

x	0	1	2	3	4
$P(x)$.10	.25	.40	.20	.05

- a. 假設所有顧客次年的購買行為將會和前一年的相同，則下一年每位顧客會把握促銷優勢而訂購的期望次數為何？
- b. 根據一份先前歷史紀錄的分析發現訂購促銷商品的平均價格是\$20，公司從每一份訂單中賺取20% 的利潤。計算明年獲利的期望值。
- c. 若執行四次促銷的固定成本被估計是\$15,000，以及每位顧客的郵寄與處理的變動成本是\$3.00。公司需要多大的顧客群以抵補促銷的成本？

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科目：統計學

系組：統計資訊學系

年級：二

4. (20%) a. 某商店九點開始營業，每小時平均來3人，試求某日早上九點到九點半之間沒有客戶上門的機率？ b. 某商店平均20分鐘來1人。試求某日開門後等候30分鐘以上才有客戶來的機率。

5. (20%)調查員隨機抽樣10個觀測值，發現 $\bar{x}=103$ 與 $s=17$ 。

($z_{0.1} = 1.28$, $t_{0.1,9} = 1.383$, $t_{0.1,10} = 1.372$)

- 在10%的顯著水準下是否有足夠的證據結論說母體平均數是小於110的？
- 假設你已知母體標準差為 $\sigma=17$ ，重做(a)小題。
- 請解釋(a)小題與(b)小題所產生的結論，有何差異？

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科目：微積分

系組：統計資訊學系

年級：二

1. (10%) Find the following limits:

(3%) (a) $\lim_{x \rightarrow 1^-} \left(\frac{x^3 - 1}{x - 1} \right)$

(3%) (b) $\lim_{x \rightarrow 1^+} \left(\frac{x^3 - 1}{x - 1} \right)$

(4%) (c) $\lim_{x \rightarrow 1} \left(\frac{x^3 - 1}{x - 1} \right)$

2. (10%) $f(x) = (x^2 + 3x + 5)^{10}$, find $\frac{d^{20}}{dx^{20}} f(x)$

3. (10%) $\int_0^2 (3x^2 + x - 5) dx$

4. (10%) $\int (x^2 + 2x - 3)^3 (x + 1) dx$

5. (10%) $\int_1^2 \frac{2 \ln x}{x} dx$

6. (10%) Find the area under the curve between the given
- x
- values:

$$f(x) = \frac{2x^2 + 12x}{\sqrt[3]{x^3 + 9x^2 + 17}}, \text{ from } x=1 \text{ to } x=3.$$

7. (10%) The second derivative of
- $f(x) = (x^2 - x + 1) \cdot (x^3 - 1)$
- can be written as

$$\frac{d^2}{dx^2} f(x) = Ax^3 + Bx^2 + Cx + D. \text{ What is } (A, B, C, D)?$$

8. (10%) A company finds that its revenue from selling
- x
- units of a product is
- $R(x) = x^2 + 500x$
- dollars. If sales are increasing at the rate of 50 per month, find the
- rate of change**
- of revenue when 200 units have been sold.

9. (10%) Find the
- equation for the tangent line to the curve**
- $y = x^2 \ln x - x^2$
- at
- $x = e$
- .

10. (10%) Given $f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$ and $\lambda > 0$, please derive the value of the

improper integral $\int_{-\infty}^{\infty} x f(x) dx$.

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科目：經濟學

系組：統計資訊學系

年級：二

一、Multiple Choice (48%)(每題2分)

1. What you give up to obtain an item is called your
 - a. explicit cost.
 - b. monetary cost.
 - c. opportunity cost.
 - d. direct cost.
2. Which of the following changes would *not* shift the demand curve for a good or service?
 - a. a change in income.
 - b. a change in the price of the good or service.
 - c. a change in expectations about the future price of the good or service.
 - d. a change in the price of a related good or service.
3. If goods A and B are complements, then an increase in the price of good A will result in
 - a. more of good B being sold.
 - b. more of good A being sold.
 - c. less of good B being sold.
 - d. no difference in the quantity sold of either good.
4. A supply curve slopes upward because
 - a. an increase in input prices increases supply.
 - b. an increase in price gives producers an incentive to supply a larger quantity.
 - c. the quantity supplied of most goods and services increases over time.
 - d. as more is produced, total cost of production falls.
5. The smaller the price elasticity of demand, the
 - a. steeper the demand curve will be through a given point.
 - b. flatter the demand curve will be through a given point.
 - c. more strongly buyers respond to a change in price between any two prices P_1 and P_2 .
 - d. smaller the decrease in equilibrium price when the supply curve shifts rightward from S_1 to S_2 .
6. Suppose the price of potato chips decreases from \$1.45 to \$1.25 and, as a result, the quantity of potato chips demanded increases from 2,000 to 2,200. Using the midpoint method, the price elasticity of demand for potato chips in the given price range is
 - a. 2.00.
 - b. 1.55.
 - c. 1.00.
 - d. 0.64.
7. When a tax is imposed on a good for which the supply is relatively elastic and the demand is relatively inelastic,
 - a. buyers of the good will bear most of the burden of the tax.
 - b. sellers of the good will bear most of the burden of the tax.
 - c. buyers and sellers will each bear 50 percent of the burden of the tax.
 - d. both equilibrium price and quantity will increase.

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系組：統計資訊學系

年級：二

8. The minimum wage is an example of a
- price ceiling.
 - price floor.
 - wage subsidy.
 - tax.
9. Total surplus is equal to
- value to buyers - profit to sellers.
 - consumer surplus x producer surplus.
 - (consumer surplus + producer surplus) x equilibrium quantity.
 - value to buyers - cost to sellers.
10. Efficiency is attained when
- total surplus is maximized.
 - producer surplus is maximized.
 - all resources are being used.
 - consumer surplus is maximized and producer surplus is minimized.
11. A difference between explicit and implicit costs is that
- explicit costs must be greater than implicit costs.
 - explicit costs do not require a direct monetary outlay by the firm, whereas implicit costs do.
 - implicit costs do not require a direct monetary outlay by the firm, whereas explicit costs do.
 - implicit costs must be greater than explicit costs.
12. A negative externality will cause a private market to produce
- less than is socially desirable.
 - more than is socially desirable.
 - exactly the quantity that is socially desirable.
 - less than the same market would produce in the presence of a positive externality.
13. The accountants hired by the Brookside Racquet Club have determined total fixed cost to be \$75,000, total variable cost to be \$130,000, and total revenue to be \$145,000. Because of this information, in the short run, the Brookside Racquet Club should
- shut down.
 - exit the industry.
 - stay open because shutting down would be more expensive.
 - stay open because the firm is making an economic profit.
14. Which of the following is a characteristic of a natural monopoly?
- Fixed costs are typically a small portion of total costs.
 - Average total cost declines over large regions of output.
 - The product sold is a natural resource such as diamonds or water.
 - All of the above are correct.
15. A market structure with only a few sellers, each offering similar or identical products, is known as
- perfect competition.
 - monopoly.
 - monopolistic competition.
 - oligopoly.

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系組：統計資訊學系

年級：二

16. The prisoners' dilemma provides insights into the
- difficulty of maintaining cooperation.
 - benefits of avoiding cooperation.
 - benefits of government ownership of monopoly.
 - ease with which oligopoly firms maintain high prices.
17. Gross domestic product measures
- income and expenditures.
 - income but not expenditures.
 - expenditures but not income.
 - neither income nor expenditures.
18. Economists use the term inflation to describe a situation in which
- the economy's overall output of goods and services is rising faster than the economy's overall price level.
 - some prices are rising faster than others.
 - the economy's overall price level is rising.
 - the economy's overall price level is high, but not necessarily rising.
19. Other things equal, relatively poor countries tend to grow
- slower than relatively rich countries; this is called the poverty trap.
 - slower than relatively rich countries; this is called the fall-behind effect.
 - faster than relatively rich countries; this is called the catch-up effect.
 - faster than relatively rich countries; this is called the constant-returns-to-scale effect.
20. Which of the following is not a function of money?
- a unit of account
 - a store of value
 - medium of exchange
 - liquidity
21. The Fisher effect says that
- the growth rate of the money supply is negatively related to the velocity of money.
 - real variables are heavily influenced by the monetary system.
 - the nominal interest rate adjusts one for one with the inflation rate.
 - All of the above are correct.
22. Which of the following correctly explains the crowding-out effect?
- An increase in government expenditures decreases the interest rate and so increases investment spending.
 - the reduction in aggregate demand that results when a fiscal expansion causes the interest rate to increase.
 - the reduction in aggregate demand that results when a monetary expansion causes the interest rate to decrease.
 - All of the above are correct.

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系組：統計資訊學系

年級：二

23. Which of the following would cause stagflation?
- aggregate demand shifts left
 - aggregate demand shifts right
 - aggregate supply shifts right
 - aggregate supply shifts left
24. According to liquidity preference theory, the money-supply curve is
- upward sloping.
 - downward sloping.
 - vertical.
 - horizontal.

二、問答與計算 (52%)

(須寫出推導或計算過程並說明理由，否則不予計分)

1. 蘋果市場僅有甲與乙二人，其需求函數分別為 $Q=10-P$ 和 $Q=14-2P$ ，
- 請問在何種情形下市場僅剩甲一人願意有需求，請說明理由。(5分)
 - 若今天市場只有一位供給者，其供給函數為 $Q=5P$ ，請問該市場的均衡數量與均衡價格為何?(10分)
 - 兩個消費者在市場均衡價格下各自購買數量為何?(6分)
 - 當市場價格為2時，市場成交量為何?此時有超額供給或超額需求?其數量為何?(6分)
2. 一簡單凱因斯開放模型， C :消費= $100+0.5(Y-T)$ 、 I :投資= 70 、 G :政府支出= 120 、 T :稅收= $90+0.2Y$ 、 EX :出口= 50 、 IM :進口= $10+0.15Y$ 。試問：
- 該經濟體系的均衡所得為多少?若充分就業 $Y_f=500$ ，此時為膨脹缺口或緊縮缺口?(13分)
 - 若要消除此缺口，政府支出應如何變動?(6分)
 - 若要消除此缺口，租稅應如何變動?(6分)

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系組：統計資訊學系

年級：三

1. (20%)統計學上有一重要性質為「不偏性」。假設隨機變數 $X_i, i=1,2,\dots,n$ 為常態分佈，也就是 $X_i \sim N(\mu, \sigma^2), i.i.d.$ 。假設 $\hat{\mu}$ 是 μ 的估計值，試問：
- 何謂不偏性？
 - 請舉例出三個具不偏性的統計量，並決定使用其中的哪一個，解釋建議選用的原因？
2. (20%)快速檢驗(Rapid Test) 被用來判斷某人是否有HIV (造成AIDS 的病毒)。錯誤陽性與錯誤陰性發生的機率分別是.027 和.080。一位醫師剛收到一份快速檢驗報告，病患檢測的結果呈現陽性。在收到此報告之前，這位醫生將這位病患歸類在低危險群，其為HIV 帶原的機率只有0.5%。這位病患實際有HIV的機率為何？此外，敏感度(Sensitivity)、特异性(Specificity)、陽性預測值與陰性預測值為何？
3. (20%)為了確定一種新品的肥料比目前使用的種類更有效，研究人員用12 塊的田地，其分散在縣市各地。每一塊地被區分為兩個大小相等的次區塊，其中之一施加目前使用的肥料，另外一塊地則施加新品肥料。在這些地上種植小麥，並且對農作產量進行測量。($t_{0.05,11} = 1.796, t_{0.05,12} = 1.782$)

田地	1	2	3	4	5	6	7	8	9	10	11	12
目前肥料	56	45	68	72	61	69	57	55	60	72	75	66
新品肥料	60	49	66	73	59	67	61	60	58	75	72	68

- 以5% 的顯著水準，能否結論說新品肥料的效果比目前使用的肥料更好？
- 以 90% 的信心水準，估計兩種肥料之間農作物產量的差異。

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科目：統計學

系組：統計資訊學系

年級：三

4. (20%)某特定品種的蕃茄種子有90%的機率會發芽。一位農夫在後院種了10粒種子。
- 恰好9粒種子發芽的機率為何？
 - 9粒或更多種子發芽的機率為何？
 - 9粒或更少種子發芽的機率為何？
 - 種子發芽的期望粒數為何？

5. (20%)迴歸分析對於成本估計是重要的應用之一。假設考慮採集六組的樣本中，生產作業之產量(X)與成本(Y)有線性關係，且樣本資料結果如下：

$$\bar{X} = 575, \bar{Y} = 5616.67; \sum_{i=1}^6 (X_i - \bar{X})^2 = 93750, \sum_{i=1}^6 (Y_i - \bar{Y})^2 = 5648333,$$

$$\sum_{i=1}^6 (X_i - \bar{X})(Y_i - \bar{Y}) = 712500$$

請問：利用樣本資料結果得到最小平方方法之迴歸線為何？在隨機誤差的變異數假設均相等下，若 $\sum_{i=1}^6 (Y_i - \hat{Y})^2 = 233333$ ，試建立出 ANOVA 表，並計算判定係數為多少？解釋判定係數在此題上意義？

n=10

二項分配機率值表

k	P														
	0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	0.904	0.599	0.349	0.107	0.056	0.028	0.006	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	0.996	0.914	0.736	0.376	0.244	0.149	0.046	0.011	0.002	0.000	0.000	0.000	0.000	0.000	0.000
2	1.000	0.988	0.930	0.678	0.526	0.383	0.167	0.055	0.012	0.002	0.000	0.000	0.000	0.000	0.000
3	1.000	0.999	0.987	0.879	0.776	0.650	0.382	0.172	0.055	0.011	0.004	0.001	0.000	0.000	0.000
4	1.000	1.000	0.998	0.967	0.922	0.850	0.633	0.377	0.166	0.047	0.020	0.006	0.000	0.000	0.000
5	1.000	1.000	1.000	0.994	0.980	0.953	0.834	0.623	0.367	0.150	0.078	0.033	0.002	0.000	0.000
6	1.000	1.000	1.000	0.999	0.996	0.989	0.945	0.828	0.618	0.350	0.224	0.121	0.013	0.001	0.000
7	1.000	1.000	1.000	1.000	1.000	0.998	0.988	0.945	0.833	0.617	0.474	0.322	0.070	0.012	0.000
8	1.000	1.000	1.000	1.000	1.000	1.000	0.998	0.989	0.954	0.851	0.756	0.624	0.264	0.086	0.004
9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.994	0.972	0.944	0.893	0.651	0.401	0.096

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科目：微積分

系組：統計資訊學系

年級：三

1. (10%) $\int_2^3 \frac{1}{(x-1)^2} dx$

2. (10%) $\int_{-1}^4 \sqrt{5+x} dx$

3. (10%) $\frac{d}{dx} \left(\frac{1-6x^2}{x^4-8x^2+7} \right)$

4. (10%) $\frac{d}{dz} [z^2 + (z^2 - 1)^3]^5$

5. (10%) $\lim_{x \rightarrow -1} \left(\frac{3x^3 - 3x^2 - 6x}{x^2 + x} \right)$

6. (10%) Given $\frac{d}{d\theta} \sin \theta = \cos \theta$, $\frac{d}{d\theta} \cos \theta = -\sin \theta$, and $\frac{d}{d\theta} \alpha \sin \theta = \alpha \cos \theta$,

please find $\frac{dy}{dx}$ for $x \cos y = \sin(x+y)$.

7. (10%) Given $f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$ and $\lambda > 0$, please derive the value of

the improper integral $\int_{-\infty}^{\infty} x^2 f(x) dx$.

8. (10%) Find the area under the curve between the given x -values:

$$f(x) = \frac{2x^2 + 12x}{\sqrt[3]{x^3 + 9x^2 + 17}}, \text{ from } x=1 \text{ to } x=3.$$

9. (10%) The demand function for a product is $p = 250 + 14x - x^2$, and the supply function for it is $p = 154 + 18x + x^2$, where p is the number of dollars and x is the number of units. If the equilibrium price is \$298, what is the producer's surplus at the equilibrium price? Round to the nearest cent.

10. (10%) $\int e^x \cos x dx$

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