(107)輔仁大學轉學生招生考試試題	考試日期：107 年 7 月 9 日第二節
	本試題共： / 頁(本頁為第 / 頁)

科目：微積分　　系組：數學系應用數學組　　年級：二

1. (12 points) Let $y = \sqrt[4]{1+\sin 5x}$. Find dy/dx.

2. (12 points) Evaluate the integral $\int \frac{\sec^2(\ln x)}{2x} dx$.

3. (12 points) Evaluate the integral $\int_1^e \sqrt{t} \ln t dt$.

4. (12 points) Find the limit $\lim_{x\to 0}(x + e^{x/3})^{3/x}$.

5. (13 points) Let $w = \ln(x+y) - \ln(x-y)$, where $x = te^s$ and $y = e^{st}$. Find $\partial w/\partial t$ and express your final answer in terms of s and t.

6. (13 points) Evaluate the iterated integral $\int_0^1 \int_0^{\sqrt{1-y^2}} \sin(x^2+y^2) dx dy$.

7. (13 points) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)2^n}$.

8. (13 points) Show that

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$$

does not exist.

※ 注意：1. 考生須在「彌封答案卷」上作答。
2. 本試題紙空白部份可當稿紙使用，試題須隨答案卷繳回。
3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

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	本試題共： / 頁(本頁為第 / 頁)

科目：微積分　　系組：數學系應用數學組　　年級：三

1. (12 points) Let $f(t) = t\sin(\pi/t)$. Find $f''(2)$.

2. (12 points) Evaluate the improper integral $\int_1^\infty 2xe^{-x^2}dx$.

3. (12 points) Let $w = e^{xy+z}$, where $x = s + t$, $y = s - t$, and $z = t^2$. Find $\partial w/\partial t$ and express your final answer in terms of s and t.

4. (12 points) Evaluate the integral $\int_{\pi/6}^{\pi/4} x\sec^2 x dx$.

5. (13 points) Show that

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

does not exist.

6. (13 points) Find the limit $\lim_{x\to 0}(\cos x)^{1/x^2}$.

7. (13 points) Evaluate the double integral $\iint_S ydA$, where S is the first quadrant polar rectangle inside $x^2 + y^2 = 4$ and outside $x^2 + y^2 = 1$.

8. (13 points) Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{n2^n}$.

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(107)輔仁大學轉學生招生考試試題	考試日期：107 年 7 月 9 日第三節
	本試題共： 1 頁(本頁為第 1 頁)

科目：線性代數　　系組：數學系應用數學組　　年級： 三

1. (12 points) In $\mathbb{R}^3$, determine if the set $\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}1\\-1\\2\end{pmatrix}\right\}$ is linearly independent.

2. (12 points) The set of all 2×2 matrices with entries from $\mathbb{R}$ is denoted by $M_2(\mathbb{R})$. Define $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ by

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+b & 0 \\ 0 & c+d \end{pmatrix}.$$

Determine whether T is a linear transformation.

3. (12 points) Find the rank and the nullity of the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -1 \end{pmatrix}$$

4. (12 points) Evaluate the determinant of the matrix

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

5. (13 points) Determine whether A is diagonalizable and, if so, find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

$$A = \begin{pmatrix} 2 & 0 & 0 & 2 \\ 0 & 3 & 2 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

6. (13 points) Use the Gram-Schmidt Process to find an orthogonal basis for the column spaces of the matrices

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

7. (13 points) Show that if A is a real symmetric matrix, then the eigenvalues of A are real.

8. (13 points) Find a change of variable that transforms the quadratic form

$$f(x_1, x_2) = 5x_1^2 + 4x_1x_2 + 2x_2^2$$

into one with no cross-product terms.

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